SJA MATHEMATICS CONTEST I

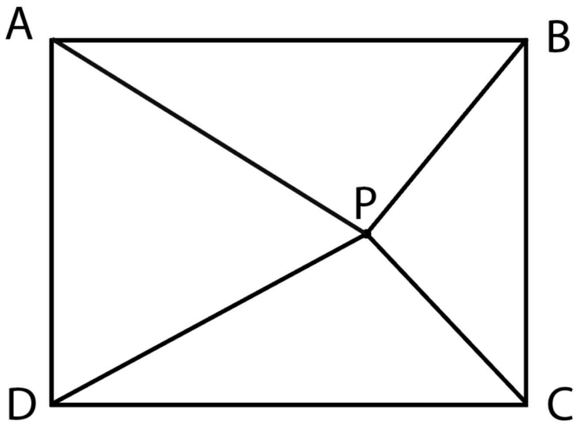
April 5, 2024

ADVANCED TEAM ROUND

1. If the greatest common divisor of $k \times 561$ and 2520 is 504, find the smallest positive integer value of k.

2. Six runners from Pokpo and six runners from Shikmoolji race against each other. During the race, four runners stumble during the race. The probability that the two are from Pokpo and the other two are from Shikmoolji, when reduced to its simplest form, is $\frac{m}{n}$. Find $m + n$. (The four runners were determined randomly)

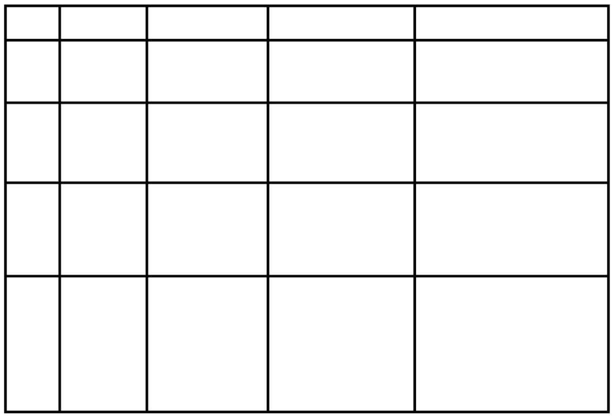
3. Point P is placed inside a rectangle $ABCD$ where $AP = \sqrt{33}$ and $DP = 5$, as shown. If the lengths of BP and CP are integers, find the difference between BP and CP.



4. Hilly can either grab 4, 5, or 6 candies at once from a jar. Then, Hilly would put the candies inside her pocket. Assuming each step is distinct, how many ways can she fill her pocket with exactly 20 candies? (Hilly never puts the candy back into the jar)

5. When x^3 is given as $1 + 7i$ where $i = \sqrt{-1}$, solve for $x^6 + x^5 - 2x^3 - 2x^2 + \frac{50}{x} + 51$.

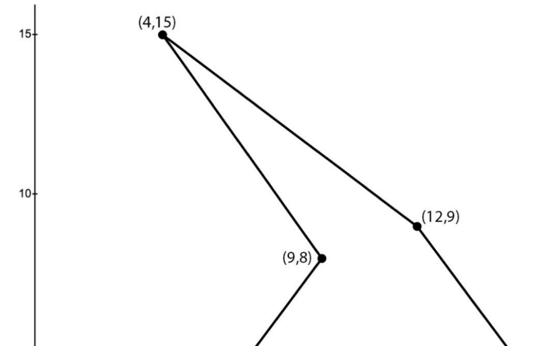
6. How many rectangles exist in the grid shown below?



7. Hilly is packing her bag for a travel. She can pick any of the following: a water bottle, a notebook, a polaroid, a headphone, a laptop, a watch, a sunscreen, and a radio. If Hilly packs at least two objects, in how many different ways can she pack her bag?

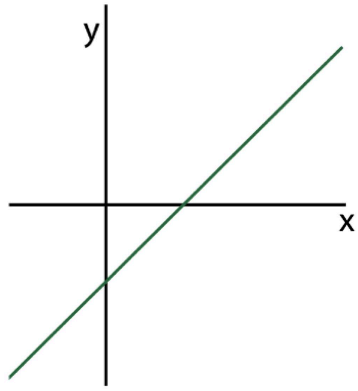
8. What is the maximum value of $a + b$ which makes both $\sqrt{125(a + 5)}$ and $\sqrt{18(b - 3)}$ positive integers? (a and b are positive integers less than 100).

9. Points (3,0), (9,8), (4,15), (12,9), (15,5) form an irregular polygon, as shown. The sum of its area and perimeter can be written as $\frac{a}{2} + \sqrt{b}$ where a and b are both positive integers. Find $a + b$.



10. $f(x)$ and $g(x)$ are given as $f(x) = -19x^2 - 9x - 1$ and $g(x) = -19x^2 - 9x + 2$. Find the value of $g(0) + g(1) + g(2) + ... + g(2024) - \{f(0) + f(1) + ... + f(2024)\}$.

11. The linear function $-ax + by = c$ is shown on the right.



Which quadrant does $y = acx - ab$ not pass through?

If your answer is …

I Quadrant, then write the answer as 1

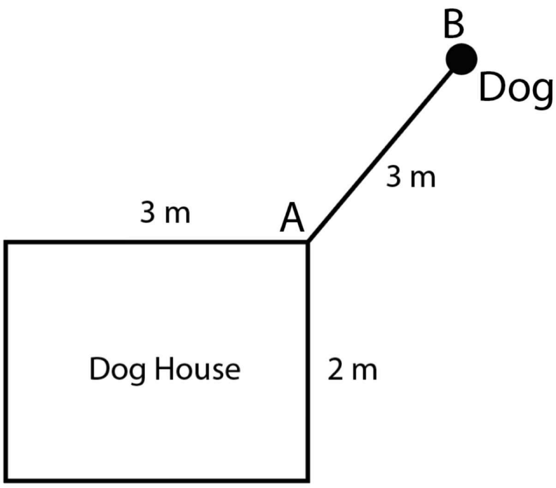
II Quadrant, then write the answer as 2

III Quadrant, then write the answer as 3

IV Quadrant, then write the answer as 4

12. N is a multiple of the sum of the divisors of 2024. If N has 64 divisors, find the smallest positive integer value of N.

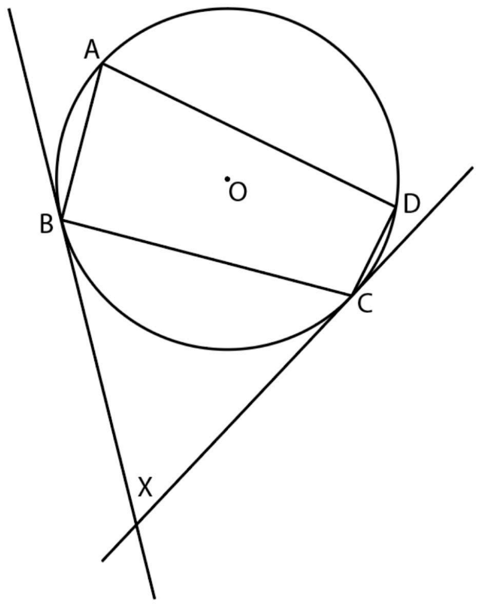
13. A dog's leash is 3m long and is connected to the outer walls of the 3m × 2m dog house, as shown. The dog can move anywhere as long as one end of the leash is connected to the outer walls of the dog house and another end is connected to the dog. The area outside of the dog house in which the dog can move around can be represented in the form $a\pi + b$, where a and b are positive integers. Find $a + b$.



14. When $2044 \times 2054 + 25 = n^2$, what is the sum of the digits of n? (n is a positive integer)

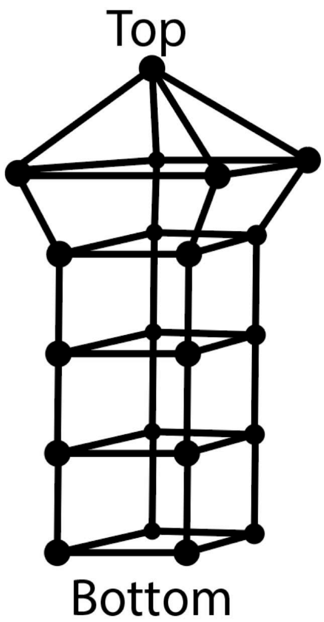
15. Complex number z is defined by $z = (x^2 - 20x + 91) + (x^2 - 10x + 21)i$, where $i = \sqrt{-1}$ and $\bar{z}$ is the complex conjugate of z. If $z \neq 0$, what os the value of x that satisfies $z = \bar{z}$?

16. Quadrilateral $ABCD$ is inscribed in a circle O, as shown. $\angle DBC = 20^\circ$ and $\angle BCD = 110^\circ$. Two lines are tangent to the circle O at B and C, respectively. The tangent lines intersect to form $\angle X$. What is the sum, in degrees, of $\angle BAC$ and $\angle X$?



17. Ms. L distributes N number of candies to 3 students: Tom, Jerry, and Teddy. She gives Tom 3 candies, Jerry 2 candies, and Teddy 1 candy. Then, she has 66 ways to distribute candies that are left to three students. How many candies were there initially?

18. The ant wants to walk down the tree house consisting of 21 vertices, as shown. If the ant only moves downward and horizontally, never revisiting any vertex, in how many ways can the ant move down the tree house? (The ant starts from the top. When the ant reaches any four vertices at the bottom, the ant stops.)



19. How many divisors do $(196 - 4 \times 14)^2 - 18(196 - 4 \times 14) - 63$ have?

20. Hilly has 18 digit number. However, Hilly spilled water on the number, and the three digits were blurred, as shown following:

If Hilly knows that the prime factorization of the number is $2^{11} \times 3^8 \times 5^4 \times 7^6 \times 11^2$ and the leftmost blurred digit is greater than the sum of the other two blurred digits, find the sum of the three blurred digits.