SJA MATHEMATICS CONTEST I

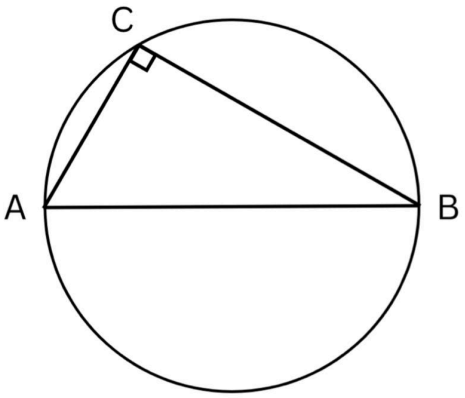
April 5, 2024

INTERMEDIATE INDIVIDUAL ROUND

1. There are 2 blue balls, 2 red balls, and 3 green balls in a box. You will choose 3 balls, and the chosen balls are never placed back into the box. What is the probability of choosing a red ball first, a blue ball second, and then a green ball?

   (A) $\frac{1}{3}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{2}{35}$ **(E)** $\frac{3}{64}$

2. A right triangle ABC with $AC = 2$ and $BC = 5$ is inscribed in a circle. What is the circumference of the circle?



   (A) 7 **(B)** 29π **(C)** $\sqrt{29}\pi$ **(D)** $\sqrt{7}\pi$ **(E)** 20

3. The difference between 102 and the sum of four positive consecutive even integers is equal to the smallest of the four integers. What is the value of the largest number among the four integers?

   (A) 20 **(B)** 24 **(C)** 28 **(D)** 32 **(E)** 36

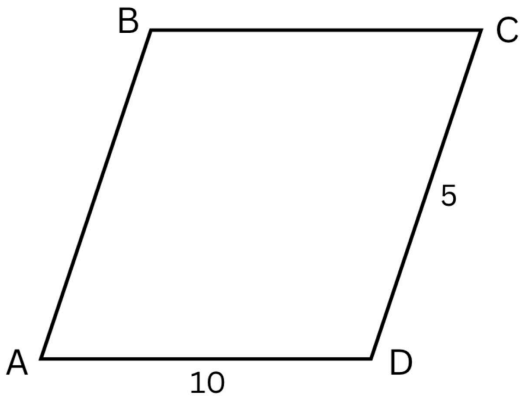
4. A cube has a side length of 4. What is the length of the longest diagonal of the cube?

   (A) $4\sqrt{3}$ **(B)** $4\sqrt{6}$ **(C)** $6\sqrt{6}$ **(D)** $6\sqrt{10}$ **(E)** $8\sqrt{2}$

5. Hilly currently has 30 pancakes. She wants to share the pancakes with two friends so that each of them, including Hilly, has a prime number of pancakes. If Hilly has the least number of pancakes, how many does she have?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

6. Parallelogram $ABCD$ has side $AB = 10$ and $CD = 5$. One of its altitudes has a length of 12. If the area of the parallelogram is less than 100, what is the length of the other altitude?



(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

7. Sam, Max, and Chloe each roll a six-sided dice once. What is the probability of them getting a sum of 6?

(A) $\frac{1}{108}$ **(B)** $\frac{5}{108}$ **(C)** $\frac{7}{108}$ **(D)** $\frac{11}{108}$ **(E)** $\frac{1}{2}$

8. What is $a + b + c$?

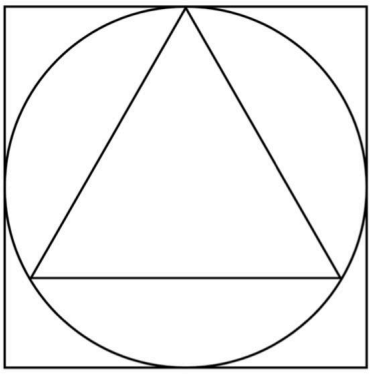
$$1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{x}}} = \frac{cx + 2}{ax + b}$$

(A) 6 **(B)** 7 **(C)** 8 **(D)** 9 **(E)** 10

9. Let n be the smallest positive integer that must be multiplied to 2024 to make it a perfect square. Find the sum of the digits of n.

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

10. A circle is inscribed in a square with a side length of 8. An equilateral triangle is then inscribed in that circle, as shown. What is the area of the triangle?



(A) $12\sqrt{3}$ **(B)** $15\sqrt{3}$ **(C)** $15\sqrt{6}$ **(D)** $15\sqrt{10}$ **(E)** $18\sqrt{3}$

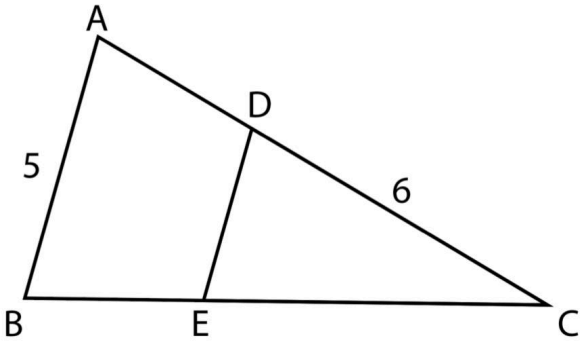
11. Hilly took four tests and the average was 89. After Hilly took another test, the average test score became 90. What is the score Hilly got in the last test?

(A) 80 **(B)** 92 **(C)** 94 **(D)** 96 **(E)** 98

12. Hilly uses a random number generator twice to choose two integers from the interval [1, 5]. What is the probability that the sum of the two numbers is odd? (The numbers are allowed to be repeated)

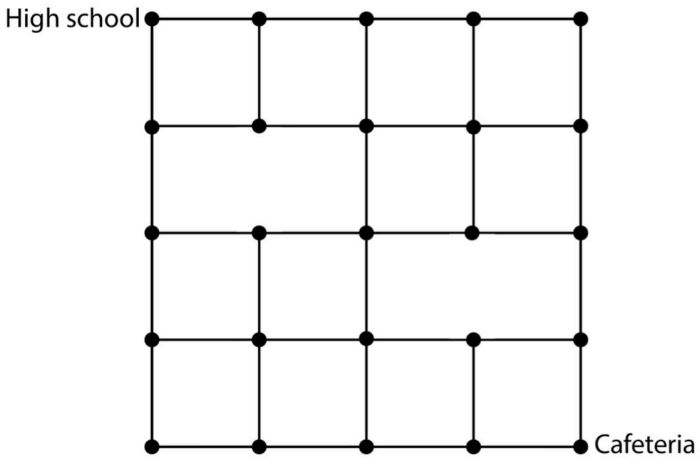
(A) $\frac{1}{25}$ **(B)** $\frac{4}{25}$ **(C)** $\frac{8}{25}$ **(D)** $\frac{12}{25}$ **(E)** $\frac{1}{2}$

13. Points D and E are on AC and BC, respectively, so that DE is parallel to AB, as shown. Triangle ABC has $AB = 5$, $DC = 6$, and $AB = DE$. What is the length of AD?



(A) $2\sqrt{13}$ **(B)** $1 + \sqrt{39}$ **(C)** $2 + 2\sqrt{39}$ **(D)** $3\sqrt{39}$ **(E)** $-3 + \sqrt{39}$

14. Hilly wants to walk down from the high school to the cafeteria. If Hilly can only move down and to the right, how many different paths exist between the high school and the cafeteria?



(A) 34 **(B)** 40 **(C)** 48 **(D)** 68 **(E)** 70

15. An operation has a property where $x \diamondsuit y \vartriangle z = \frac{y}{xz} + z$ for all positive integers x, y, and z. The maximum value of $a \diamondsuit b \vartriangle c + b \diamondsuit c \vartriangle a + a \diamondsuit c \vartriangle b$, when reduced to its simplest form, is $\frac{\alpha}{\beta}$. Find $\alpha + \beta$.

(A) 112 **(B)** 159 **(C)** 217 **(D)** 281 **(E)** 325