

SJA MATHEMATICS CONTEST II

April 11, 2025

ADVANCED INDIVIDUAL ROUND

1. Three six-sided dice are rolled. The probability that the product of the three numbers is 24 is, in its simplest form, $\frac{a}{b}$. What is $a + b$?

(A) 27 (B) 49 (C) 63 (D) 77 (E) 112

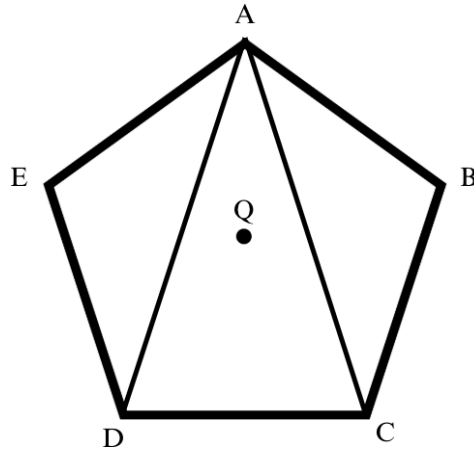
2. How many ways can we arrange the letters in the word "COMBINATORICS" such that all the vowels (a, e, i, o, u) appear before all the consonants?

(A) 151200 (B) 302400 (C) 604800 (D) 1209600 (E) 2419200

3. Hilly is time-traveling. Hilly started time travel on Wednesday, October 23, 2024, and jumped 1000 days into the future. On what date will Hilly arrive? (1 year = 365 days)

(A) July 20 (B) June 30 (C) August 1 (D) July 22 (E) July 21

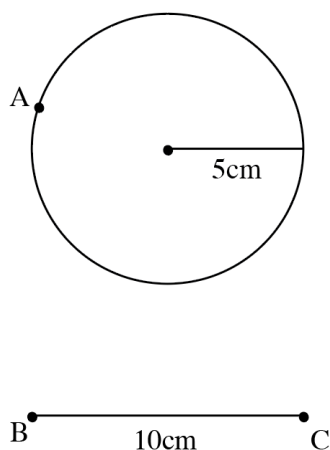
4. Pentagon $ABCDE$ is a regular pentagon, and the point Q is equidistant from the vertices. Find the angle $\angle ADQ$.



- (A) 18 (B) 19 (C) 20 (D) 21 (E) 24
5. When $x = 1 + i$ and $y = 1 - i$, solve for $x^5 + y^5$.

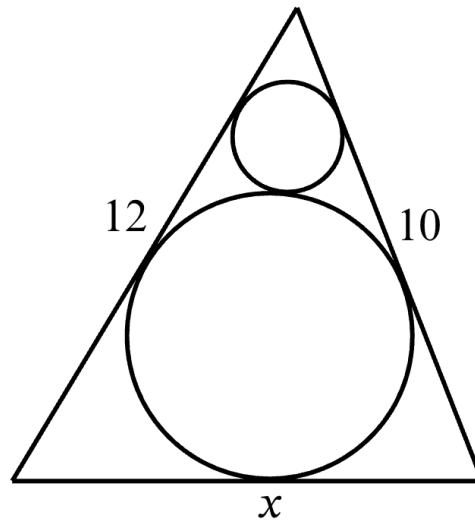
- (A) 8 (B) 4 (C) 1 (D) -4 (E) -8

6. A circle in the diagram has a radius of 5 cm, and its center is 10 cm away from the line segment BC , which has a length of 10 cm. A point A on the circle is 11 cm away from the line segment BC . Starting from the given position in the diagram, point A moves clockwise around the circle at a constant speed of one full revolution every 2 minutes. The area of the triangle formed by connecting points A , B , and C is measured once every 15 seconds, for a total of 8 times. What is the total sum of 8 triangle ABC areas?



- (A) 250 (B) 300 (C) 350 (D) 400 (E) 450
7. In Hilly's Math Contest, you have to choose to answer a total of 12 questions from Part A and Part B and you must answer at least 5 from each part. There are 7 questions in Part A and 9 questions in Part B. How many ways could the 12 questions be answered?
- (A) 1450 (B) 1470 (C) 1600 (D) 1720 (E) 1740
8. Given $a + 3ab + b = 10$, find $a + b$ where a and b are both non-negative integers.
- (A) 9 (B) 10 (C) 11 (D) 12 (E) 13

9. The radius of the larger inscribed circle is $\frac{\sqrt{10}}{2} + 1$ and the radius of the smaller inscribed circle is $\frac{\sqrt{10}}{2} - 1$. Find x .



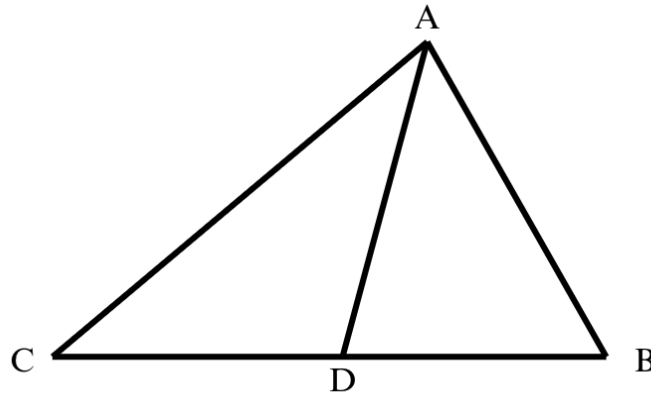
- (A) $6\sqrt{3}$ (B) 11 (C) 12 (D) $12\sqrt{2}$ (E) 14
10. When $f(\alpha) = f(\alpha - 1) - f(\alpha - 2)$, what is the value of $f(2025)$? $f(1)$ and $f(2)$ are given as 3 and 5, respectively.

- (A) 0 (B) 1 (C) 2 (D) 4 (E) 8

11. Hilly starts at the bottom-left corner of a 6 x 6 grid and moves only right or up. However, it must pass through the point (3,3) at some point before reaching the top-right corner. What is the probability that a randomly chosen path satisfies the condition mentioned?

- (A) $\frac{100}{234}$ (B) $\frac{100}{233}$ (C) $\frac{100}{231}$ (D) $\frac{200}{461}$ (E) $\frac{100}{229}$

12. In triangle ABC , $\angle CAB = 60^\circ$. Line AD which has a length of $x + 2$ dissects $\angle CAB$. The length of line AC is $2x + 4$ and the length of line BD has a length of $3\sqrt{7}$. $ax^2 + bx - c$ expresses the difference between the area of triangle ACD and triangle ABD in terms of x given that $x > 1$. Find $a + b - c$.



- (A) $\sqrt{2}$ (B) 2 (C) 3 (D) $4\sqrt{2}$ (E) 8
13. In Hilly's high school, there are a total of 300 students. Out of those 300 students, 31% are varsity soccer players, 54 students are varsity volleyball players, and 9% are playing both. What is the probability of choosing a student who doesn't play any sports?
- (A) $\frac{3}{14}$ (B) $\frac{3}{10}$ (C) $\frac{3}{8}$ (D) $\frac{3}{7}$ (E) $\frac{3}{5}$
14. A triangle is formed by the region bounded by $y_1 = ax$, $y_2 = -ax + b$, and the x-axis. If the area of this triangle is 8 and $0 < b \leq 8$. What is the maximum possible value of a ?

- (A) $\sqrt{2}$ (B) 2 (C) 3 (D) $4\sqrt{2}$ (E) 8

15. When x and y are real numbers,
 $(\log y)^2 + (2^{x+1} + 2^{-x+1}) \cdot \log y + 2^{2x+1} + 2^{-2x+1} = 0$. Find the value of
 $1000x + 1000y$.

- (A) 0 (B) 10 (C) 100 (D) 1000 (E) 10000