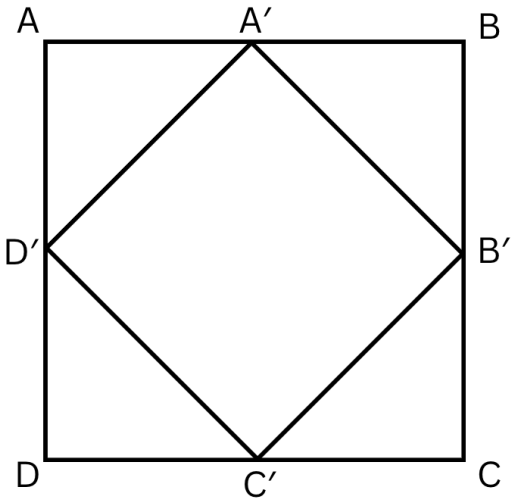
SJA MATHEMATICS CONTEST II

April 11, 2025

INTERMEDIATE INDIVIDUAL ROUND

1. A square $ABCD$ has side length of 10. Another square $A'B'C'D'$ is drawn by connecting the midpoints of each side of square $ABCD$. What is the difference between the area of $ABCD$ and area of $A'B'C'D'$?



Answer: 50

Explanation: Area of ABCD is 100, Area of A'B'C'D' is 50 because it is half of ABCD. Difference is $100 - 50 = 50$.

2. Only one of the four digits of the number 2025 is odd. How many integers between 1000 and 9999, inclusive, have the property that only one digit is odd? (Digits do not need to be distinct)

Answer: 2125

Explanation: We have to consider two cases: the first digit is odd or one of the other digits is odd. If the first digit is odd, there are 5 possible digits (1,3,5,7,9) for the first digit. The remaining 4 digits should be even, therefore, there are 5 possibilities for each (0,2,4,6,8).

# of possible ways = $5 * 5 * 5 * 5 = 625$

If the second is odd, there are 4 possible digits (2,4,6,8) for the first digit since the first digit could not be 0. The second digit would have 5 options (1,3,5,7,9) and the remaining digits would also have 5 options each (0,2,4,6,8).

# of possible ways = $4 * 5 * 5 * 5 = 500$

The number of integers with an odd number in the second digit is the same for the occasion where the third or fourth digit is odd. So, the total number of integers that have the property that only one digit is odd is $625 + 500 + 500 + 500 = 2125$

3. Hilly is invited to a video-call fan sign event. Hilly's favorite idol group has a total of 11 members and he has 2 biases: Billy and Rilly. However, he could only do a video call with 3 members. What is the probability of Hilly doing a video call with his two biases?

   Answer: $\frac{9}{165}$

   Explanation: To solve this problem, we have to calculate the total possible ways of doing the video call and the possible ways of doing the video call with Billy and Rilly.

   Total possible way of doing video call = 11 C 3 = 165

   Total possible way of doing a video call with Billy and Rilly = 9 C 1 = 9

   So, the probability of Hilly doing a video call with Billy and Rilly = 9/165

4. The point $A(\frac{10}{3}, a)$ is on the straight line $y = 3x$. The points B, C, and D are reflections about the x-axis, y-axis, and origin, respectively to point A. Find the area of the triangle BCD.

   Answer: $\frac{200}{3}$

   Explanation:

   As $A(\frac{10}{3}, a)$ is on the line $y = 3x, a = 3 * \frac{10}{3} = 10$

   $A(\frac{10}{3}, 10), B(\frac{10}{3}, -10), C(-\frac{10}{3}, 10), D(-\frac{10}{3}, -10)$

   As a result, $\Delta BCD = \frac{20}{3} * 20 * \frac{1}{2} = \frac{200}{3}$.

5. Given $x = \frac{1}{n} - \frac{1}{n+1} + \frac{1}{n+1} - \frac{1}{n+2} + \frac{1}{n+2} - \frac{1}{n+3} + \ldots - \frac{1}{n+50}$, what is the value of x when $n = 5$?

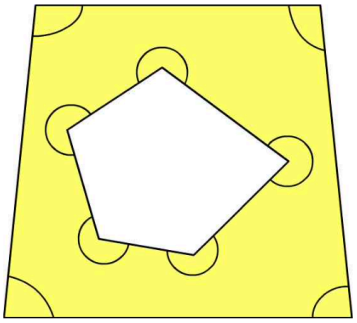
   Answer: 2/11

   Explanation:

   1/n - 1/n+50

   ⅕ - 1/55 = 2/11

6. What is the sum of all 9 angles of the yellow polygon in the diagram?



Answer: 1620

Explanation: The outer four angles have sum of 360, just like a normal quadrilateral. Before the inner pentagon is cut, the inner 5 angles have sum of 360 * 5 = 1800 degrees. However, when pentagon is cut, 540 degrees disappear, so the sum of rest five angles is 1800 - 540 = 1260. Sum of all 9 angles is 360 + 1260 = 1620.

7. In KISMC, Hilly earns 4 points for correct answers and loses 2 points for incorrect answers. There are a total of 15 questions, and if Hilly answered all the questions and received 36 as his score, what is the difference between the number of correct answers and the number of incorrect answers?

Answer: 7

Explanation:

Let x be the number of questions answered correctly and y be the number of questions answered incorrectly.

Hilly answers a total of 15 questions, therefore, $x + y = 15$.

Since Hilly earns 4 points for correct answers and loses 2 points for incorrect answers, $4x - 2y = 36$.

By using eliminating technique for those two equations, we are able to find one of the values.

When we subtract the second equation from the first equation, $y = 4$. Plugging the y-value into the first question, $x + 4 = 15$, $x = 11$.

So, the difference between the number of correct answers and the number of wrong answers is $11 - 4 = 7$

8. Three six-sided dice are rolled. The probability that the product of the three numbers is 24 is, in its simplest form, $\frac{a}{b}$. What is $a + b$?

   Answer: 77

   Explanation: Because total number of possible outcome is 6^3 and the possible ways for the product to be 24 are sets (1,4,6), (2,2,6) and (2,3,4), we consider these three choices.

   # of ways for (1,4,6) to occur : 3! = 6 ways

   # of ways for (2,2,6) to occur : 3!/2! = 3 ways

   # of ways for (2,3,4) to occur : 3! = 6 ways

   So, a total number of ways for the product to be 24 is $6 + 3 + 6 = 15$, and the probability is $\frac{15}{6^3} = \frac{5}{72}$. Therefore, $a + b$ is $5 + 72 = 77$

9. What is the digit of the 2025th decimal place of the fraction 1/7?

   Answer: 2

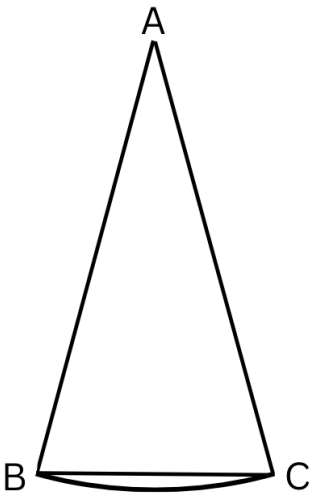
   Explanation:

   $\frac{1}{7} = \overline{14287}$

   $\frac{2025}{6} = 337\ldots 3$

   The third repeating decimal of $\frac{1}{7}$ is 2.

10. Triangle ABC is an isosceles triangle with $AB = AC$. Also, arc BC is part of a larger circle O, as shown. If $\angle BAC = 30°$ and $AB = 6$, what is the area between the circular arc BC and the triangleABC?



Answer: $3\pi - 9$

Explanation: By using the trig rules, the distance between point B and segment AC is 3 by pythagoras theorem. By using that the area of triangle = $\frac{1}{2}bh$. The area of the triangle is 9. Area of circular sector = $6^2 \cdot \pi \cdot \frac{30}{360} = 3\pi$, so the area between the circular sector and the triangle is ultimately $3\pi - 9$.

11. When three-digit natural numbers m and n make $\sqrt{2m}\ and\ \sqrt[3]{3n}$ integers, solve for a minimum of $m + n$.

Answer: 371

Explanation:

To make $\sqrt{2m}\ and\ \sqrt[3]{3n}$ a natural number, $m = 2 * k^2$ and $n = 3^2 * p^3$.

As m and n are given as three-digit integers,

$m = 2 * k^2 = 2 * 8^2 = 128$

$n = 3^2 * p^3 = 3^2 * 3^3 = 243$

Minimum of $m + n = 128 + 243 = 371$

12. In Hilly's high school, there are a total of 300 students. Out of those 300 students, 31% are varsity soccer players, 54 students are varsity volleyball players, and 9% are playing both. What is the probability of choosing a student who doesn't play any sports?

Answer: 3/5

Explanation:

# of varsity players = $0.31 * 300 = 93$

# of varsity volleyball players $= 54$

# of players playing both sports = $0.09 * 300 = 27$

#of students who don't play any sports = $300 - 93 - 54 + 27 = 180$

Therefore, the probability of choosing a student who doesn't play any sports is $\frac{180}{300} = \frac{3}{5}$

13. In Hilly's Math Contest, you have to choose to answer a total of 12 questions from Part A and Part B and you must answer at least 5 from each part. There are 7 questions in Part A and 9 questions in Part B. How many ways could the 12 questions be answered?

    Answer: 1470

    Explanation: There are three ways to answer the questions: 5 Part A and 7 Part B, 6 Part A and 6 Part B, and 7 Part A and 5 Part B.

    For the first case, there are $7\ C\ 5 = 21$ ways to choose in Part A and $9\ C\ 7 = 36$ ways to choose in Part B. Therefore, the # of ways to answer for Case 1 is $21 * 36 = 756$.

    For the second case, there are $7\ C\ 6 = 7$ ways to choose in Part A and $9\ C\ 6 = 84$ ways to choose in Part B. Therefore, the # of ways to answer for Case 2 is $7 * 84 = 588$.

    For the last case, there are $7\ C\ 7 = 1$ way to choose in Part A and $9\ C\ 5 = 126$ ways to choose in Part B. Therefore, the # of ways to answer for Case 3 is $1 * 126 = 126$.

    The total number of ways to answer the 12 questions is $756 + 588 + 126 = 1470$ ways.

14. Hilly is time-traveling. Hilly started time travel on Wednesday, October 23, 2024, and jumped 1000 days into the future. On what date will Hilly arrive? (1 year = 365 days)

    Answer: Tuesday, July 20, 2027.

    Explanation:

    1000/7=142...6

    Tuesday

    1000 days = 2 years and 270 days

After two years: October 23, 2026

8 + 30 + 31 days later → 12/31, 2026

270-69 = 201

Count backward from December 12/31, 2027

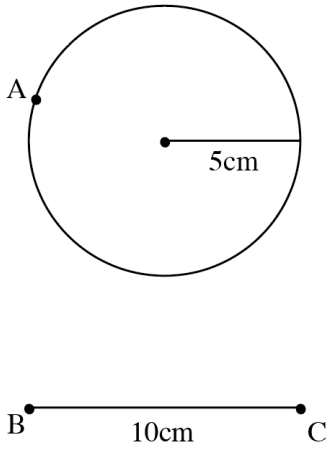
365 - 201 = 164

164 = 31 + 30 + 31 + 30 + 31 + 11

Going backward 11 days from July 31, 2027

Hilly arrives at July 20, 2027.

15. A circle in the diagram has a radius of 5 cm, and its center is 10 cm away from the line segment BC, which has a length of 10 cm. A point A on the circle is 11 cm away from the line segment BC. Starting from the given position in the diagram, point A moves clockwise around the circle at a constant speed of one full revolution every 2 minutes. The area of the triangle formed by connecting points A, B, and C is measured once every 15 seconds, for a total of 8 times. What is the total sum of 8 triangle ABC areas?



Answer: 400

Explanation: It doesn't matter where point A is if point A rotates one full circle in 2 minutes and triangle area is measured every 15 seconds. One triangle will be measured with the height of a, and 1 minute later another triangle will be measured with the same base and height of (20 - a). They will average to a triangle with height of 10, so just sum up 8 triangles with base of 10cm and height of 10cm, which gives 400.